May 8, 2020

Errata for *Vector and Geometric Calculus* Printings 1-5

p. 81. Replace the multivector G with the vector $\mathbf{g}$ and the multivector $\bar{G}$ with the vector $\bar{\mathbf{g}}$ everywhere. And in the statement of Theorem 5.27, $\boldsymbol{\partial} \cdot \mathbf{h} \to \mathbf{h} \cdot \boldsymbol{\partial}$.

p. 155, Exercise 10.17. Replace with
Let $F(x,y) = u(x,y) + v(x,y)\,\mathbf{i}$, where u and v are scalar valued and $\mathbf{i}$ is the unit pseudoscalar of the xy-plane. Suppose that F is analytic. Prove the *Cauchy-Riemann equations*: $u_x - v_y = 0$ and $v_x + u_y = 0$.

Errata for *Vector and Geometric Calculus* Printings 1-4

Note: "p. m (n)" refers to page m of Printing 4 and page n of Printings 1-3.

p. 30, proof of Theorem 3.8. "the last sum in Eq (3.6) approaches 0." $\to$ "the right side of Step (3), divided by $|\mathbf{h}|$, approaches 0 with $\mathbf{h}$."

p. 31 (29), just before Theorem 3.10. $\mathbf{f}'_\mathbf{x}(\mathbf{h}) = [\mathbf{f}'_\mathbf{x}][\mathbf{h}] \;\to\; [\mathbf{f}'_\mathbf{x}(\mathbf{h})] = [\mathbf{f}'_\mathbf{x}][\mathbf{h}]$.

p. 35 (33), proof of Theorem 3.14. (Problem 8.1.14) $\to$ (LAGA Problem 8.1.14).

p. 44, bottom. "for $\mathbf{y}(\mathbf{x})$ near $\mathbf{a}$" $\to$ "for $\mathbf{y}$ in terms of $\mathbf{x}$ near $\mathbf{a}$".

p. 56 (54), an omission, not an error. New second paragraph after Definition 4.5: "The tangent space is a vector space (LAGA Exercise 8.12)."

p. 58 (55), caption of Figure 5.14. "$\mathbf{f}'$ maps" $\to$ "$\mathbf{f}'_\mathbf{p}$ maps".

p. 62 (58), Definition 5.2. Add a footnote after the first line: "i.e., the scalar coefficients of F are differentiable."

p. 64, Problem 5.2.10. Replace "$\mathbf{f}'^{*}_\mathbf{x}(\mathbf{a})$" with "$\mathbf{f}'_\mathbf{x}(\mathbf{a})$".

p. 79 (73), Problem 5.5.1a. "Eq. (6.25)" $\to$ "Eq. (5.19)".

p. 85, first line of the proof of Theorem 6.5, improved:
"First, $\partial_i f(\mathbf{x}) h_i = \boldsymbol{\nabla} f(\mathbf{x}) \cdot \mathbf{h} = 0$."

p. 86, Definition 6.3. In "a *strict local minimum* at $\mathbf{x}$ if $f(\mathbf{x}) \le f(\mathbf{x}+\mathbf{h})$" change to "$f(\mathbf{x}) < f(\mathbf{x}+\mathbf{h})$".

p. 96, Exercise 7.1. The approximate value of the integral is 0.2570120954.

p. 99, Problem 7.1.1. At end: "Parts (c)-(d)".

p. 105. Remove Problem 7.2.9. It is a duplicate of Problem 7.2.3.

p. 115, Problem 7.2.3. Integral should read $\int_{[a,b]} \sqrt{r'^2 + r^2}\, d\theta$.

p. 129, just before Figures 8.2-8.4. Strike "to a scalar".

p. 131, Problem 9.1.4. Displayed equation should read $2\pi \int_a^b \sqrt{1+r'(z)^2}\, r(z)\, dz$.

p. 132. The formula for area in the middle of the page should be $\iint_A |\mathbf{x}_u(u,v) \wedge \mathbf{x}_v(u,v)|\, dA$.

p. 138, Problem 9.2.2. The answer is $-4\pi/3$.

p. 142, last paragraph, first sentence. "multiple integrals" $\to$ "directed integrals".

p. 144, Exercise 10.2. Replace the hint with "Fact: The boundary of a boundary is empty: $\partial(\partial M) = \varnothing$."

p. 147, improved version of

Corollary 10.3 (Generalized divergence theorem). Let M be a bounded m-dimensional manifold in $\mathbb{R}^m$ and $\bar{\mathbf{n}}$ be its outward normal. Let F be a multivector field on M. Set $d\boldsymbol{\sigma} = \bar{\mathbf{n}}\, d^{m-1}x$. Then

$$\int_M \boldsymbol{\nabla} \cdot F\, d^m x = \oint_{\partial M} d\boldsymbol{\sigma} \cdot F.$$

Proof. Suppose first that F is of a single grade, g. Use the extended fundamental identity (LAGA Theorem 6.28) to split the geometric product in both integrands of Eq. (10.5) into inner and outer products, of grades $g-1$ and $g+1$ respectively. Equating the inner product parts gives Eq. (10.7) for a single grade F.

To finish, apply this to each grade of a multigrade F and add. □

p. 153, first displayed equation: $(\boldsymbol{\nabla} \times \mathbf{f})(\mathbf{x}^*) \cdot \widehat{\mathbf{n}} \to \big((\boldsymbol{\nabla} \times \mathbf{f}) \cdot \widehat{\mathbf{n}}\big)(\mathbf{x}^*)$. And the left side of Eq. (10.14) should read: $\big((\boldsymbol{\nabla} \times \mathbf{f}) \cdot \widehat{\mathbf{n}}\big)(\mathbf{x}_0)$.

p. 166, Theorem 11.8. "Let $\mathbf{x}(u(t),\, v(t))$" $\to$ "Let $\mathbf{x}(u_1(t),\, u_2(t))$".

Errata for *Vector and Geometric Calculus* Printings 1-3

p. 30, Theorem 3.10, first line of proof:
$\big(\partial_i f_1(\mathbf{x}), \ldots, \partial_i \mathbf{f}_n(\mathbf{x})\big) \ \rightarrow\ \big(\partial_i f_1(\mathbf{x}), \ldots, \partial_i \mathbf{f}_m(\mathbf{x})\big)$.

p. 40, just before the problems. "series of a vector valued function **f** centered" $\rightarrow$ "series of an f centered".

p. 44, first line. "differentiable" $\rightarrow$ "continuously differentiable"

p. 51, Theorem 4.3 should read:
Let $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ parameterize curves and suppose that $\mathbf{x}_1'(t)$ and $\mathbf{x}_2'(t)$ exist. Then $\big(\mathbf{x}_1(t)\mathbf{x}_2(t)\big)'$ exists and

$$\big(\mathbf{x}_1(t)\mathbf{x}_2(t)\big)' = \mathbf{x}_1'(t)\,\mathbf{x}_2(t) + \mathbf{x}_1(t)\,\mathbf{x}_2'(t).$$

Change Eq. (4.6) similarly. Remove the paragraph following the theorem.

p. 55, change bottom to

Let **f** be a 1-1 map between manifolds of equal dimension.
Then the tangent map $\mathbf{f}_\mathbf{p}'$ maps $\mathsf{T}_\mathbf{p}$ to $\mathsf{T}_{\mathbf{f}(\mathbf{p})}$.

p. 58. Remove Problem 4.3.6 Part (b) and restate Part (a):
The definition of the determinant of a linear transformation *on* a vector space (LAGA Definition 8.21) does not apply to the linear transformation $\mathbf{f}_\mathbf{p}'$ because it is between *two* vector spaces. Define $\det(\mathbf{f}_\mathbf{p}')$. *Hint*: See LAGA Definition 8.21.

p. 64, Problem 5.2.11. $\boldsymbol{\nabla} \wedge \mathbf{e} = -\partial_t \mathbf{B} \ \rightarrow\ \boldsymbol{\nabla} \wedge \mathbf{e} = \partial_t \mathbf{B}$.

p. 76, Problem 11.2.7. New Part (b): Define a directional derivative for fields defined on a surface by $\partial_\mathbf{h}\mathbf{f}(\mathbf{p}) = (\mathbf{h} \cdot \boldsymbol{\partial})\mathbf{f}(\mathbf{p})$ (Definition 11.10). Compute $\partial_\mathbf{t}\mathbf{t}$ on the equator. *Ans.* $-(\sin\theta\mathbf{i} + cos\theta\mathbf{j})/\rho$.
Note that at the equator $\mathbf{t}$ is in the tangent plane but $\partial_\mathbf{t}\mathbf{t}$ is not.

p. 82, Theorem 6.7 statement. $g(\mathbf{x}_0) = c \ \rightarrow\ g(\mathbf{x}) = c$.

p. 105, Problem 5.4.1. Field should be $\mathrm{e}^x\big(\sin(xy) + y\cos(xy)\big)\mathbf{i} + x\mathrm{e}^x\cos(xy)\mathbf{j}$

p. 116, Exercise 8.8. The answer is $\pi(\mathrm{e}^4 - 1)$.

p. 132, Corollary 10.3. "Let **f** be a multivector field" $\rightarrow$ "Let **f** be a vector field"

p. 134, Problem 10.2.6c. $\displaystyle\oint_C \mathbf{e} \cdot d\mathbf{s} = \partial_t \iint_S \mathbf{B} \cdot d\mathbf{S} \ \rightarrow\ \oint_C \mathbf{e} \cdot d\mathbf{s} = -\partial_t \iint_S \mathbf{B} \cdot d\mathbf{S}$

p. 186. Add "65" and "66" to gradient entry. Add "98" to curl entry. Add "124" to divergence entry.

Errata for *Vector and Geometric Calculus* Printings 1-2

p. 14, line 8: Delete "in".

p. 23, line 4: Change the period after "tangents" into a comma.

p. 28, Exercise 3.9: Change "Eq. (3.23)" to "Eq. (3.18)".

p. 30, Exercise 3.13. $\mathbf{f}\colon U \subseteq \mathbb{R}^n \to \mathbb{R}^m \quad \to \quad \mathbf{f}\colon \mathbb{R}^n \to \mathbb{R}^m$.

p. 38, Problem 3.4.12c. Change to "The *ideal gas law* is $pv = nRT$, where n, p, v, T are the number of moles, pressure … ."

p. 43, last two lines. $\mathbf{f}(\mathbf{a}, \mathbf{b}) = \mathbf{0} \to \mathbf{f}(\mathbf{a}, \mathbf{b}) = (2, 2)$.

p. 49, Definition 4.1.
"Let $\mathbf{q} \in A$ and set $\mathbf{p} = \mathbf{x}(\mathbf{q})$." $\to$ "Let $t \in A$ and set $\mathbf{p} = \mathbf{x}(t)$."

p. 55, Figure 5.14, caption. "onto S" $\to$ "to S"

p. 59, Corollary 5.7. $\mathbf{f}'^*(\mathbf{b}) \to \mathbf{f}'^*_{\mathbf{x}}(\mathbf{b})$.

p. 59, line -5: Change "Eq. (5.2) and Eq. (3.6)" to "Eq. (5.2) and Eq. (3.23)".

p. 64, bottom. Problem 4.3.12 $\to$ LAGA Problem 4.3.12

p. 69, Eq. (5.17). "$\frac{\partial x_i}{\partial w_j}\mathbf{e}_i$" $\to$ "$\frac{\partial x_\ell}{\partial w_j}\mathbf{e}_\ell$".

p. 72, Exercise 5.36. Equations should read

$$\widehat{\boldsymbol{\phi}} = \cos\phi(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) - \sin\phi\mathbf{k}, \quad \widehat{\boldsymbol{\theta}} = -\sin\theta\mathbf{i} + \cos\theta\mathbf{j}.$$

p. 75, footnote. $\mathbb{R}^3 \to \mathbb{R}^n$.

p. 76, Problem 5.6.1b, second printing only. "even though *hat***t** is" $\to$ "even though $\widehat{\mathbf{t}}$ is".

p. 79, Theorem 6.5. A *much* better proof:
Proof. Since $\boldsymbol{\nabla} f(\mathbf{x}) = \mathbf{0}$, $\partial_i f(\mathbf{x}) h_i = 0$. And $\partial_{ij} f(\mathbf{x}) h_i h_j > 0$ for $\mathbf{h} \neq \mathbf{0}$, since $\mathbf{H} f(\mathbf{x})$ is positive definite. Then $\partial_{ij} f(\mathbf{x} + t^*\mathbf{h}) h_i h_j > 0$ for small $t^*\mathbf{h} \neq 0$, since the partial derivatives are continuous at $\mathbf{x}$. The theorem now follows from Eq. 3.2. □

p. 81, Problem 6.1.2c. $\lim_{(x,y)\to\infty} \quad \to \quad \lim_{(x,y)\to\infty} f(x, y)$.

p. 90, equation mid-page, $\int_{[a,b]} af\,dx = a\int_{[a,b]} f\,dx \to \int_{[a,b]} cf\,dx = c\int_{[a,b]} f\,dx$.

p. 91, after the sentence beginning with *Think of.* "Divide C into infinitesimal parts. Multiply the value of F on each part by the infinitesimal length ds of the part. Add to form the integral."

p. 96, note at the bottom of the page. "here here" $\to$ "here".

p. 98, Problem **??**c. Misplaced ")": "$F\big(\mathbf{x}(u, v)\,\mathbf{x}_u(u, v)\big)$" $\to$ "$F\big(\mathbf{x}(u, v)\big)\,\mathbf{x}_u(u, v)$".

p. 100, Exercise **??**. "Theorem 7.11" $\to$ "Theorem 7.10".

p. 100, proof of Theorem 7.13: 'it independent" $\to$ "it is independent".

p. 101, Exercise 5.20b. “not conservative” $\to$ “not conservative in $\mathbb{R}^2 - \{\mathbf{0}\}$”.

p. 102, following Definition 5.18. “All simple closed curves” $\to$ “All closed curves”.

p. 103, line -7. Switch “m” and “M”.

p. 103, line -5: Change “Eq. (7.10)” to “(Eq. (7.14))’.

p. 107, line -3: Change “set open” to “open set”.

p. 112, Exercise 8.2. $\int_{y=0}^{1} \to \int_{y=0}^{2}$.

p. 124, Problem 9.2.3: “scalar + trivector” $\to$ “vector + trivector”.

p. 131, Fig. 10.6: Arrows should be reversed, as the M_i are “oriented clockwise”.

p. 136, second line of the proof of Corollary 10.5: $(-1)^{2\times 2} \to (-1)^{2\times 1}$.

p. 142, below Corollary 10.10. $f(x) = \int_a^x f'(t)dt + f(a)$.

p. 146, lines 12 and 17: Change “Theorem 4.3b” and “Theorem 4.3” (both) to “Eq. (4.6)”.

p. 148, Theorem 11.5, Proof. $\bar{\dot{\mathbf{x}}} \to \dot{\mathbf{x}}$, twice

p. 151, line above Def 11.10: Change “Definition 5.23” to “Definition 5.15”.

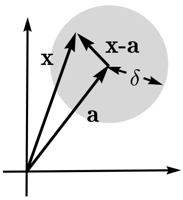
p. 158, middle displayed line in the proof of Theorem 11.25. Drop the middle term.

p. 159, line Exercise 11.25. Change “Exercise 4.12b” to “Exercise 4.12c”.

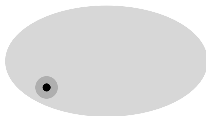
p. 162, line 5. Delete “is”.

Errata for *Vector and Geometric Calculus* Printing 1

Due to publisher error the shading in several of figures is washed out in some copies of the book. The correct shading is shown below. I think that if Figure 2.1 is OK in your book, then all figures are.



2.1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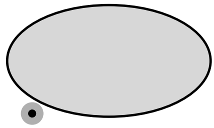
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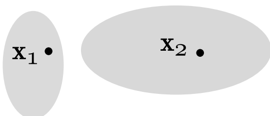

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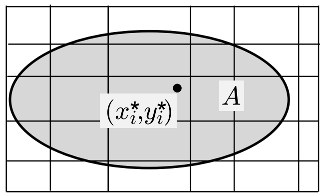
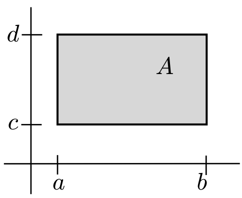


8.1



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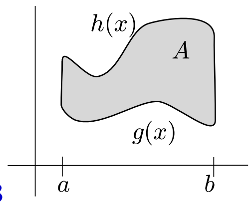

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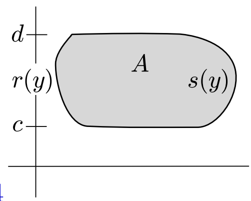

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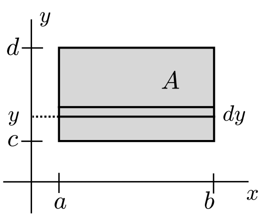
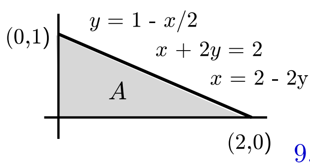
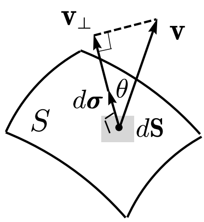


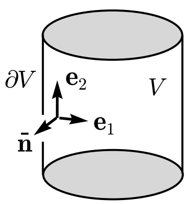
8.5



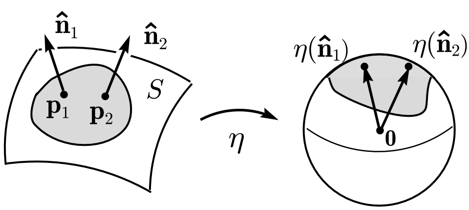
8.6



9.5



10.1



11.3

p. 4, Exercise 1.1. "$c_{ij} \to c_{ik}$".

p. 6, Problem 1.1.1. "$\mathbf{x}(t) \to \mathbf{x}(\theta)$".

p. 8, Exercise 1.11. Delete "We will do this often." Add "This will allow us to specialize formulas for surfaces defined parametrically to surfaces defined by $z = f(x, y)$. Exercise 5.38 is an example."

p. 11, Exercise 1.14. "$x \neq 0$" → "$x > 0$".

p. 12. Spherical from cylindrical transformation.
$\phi = \arccos(z/r) \to \phi = \arctan(r/z)$.

p. 22, bottom. Remove " The definition shows that $\partial_i F$ has the same grades as F." Parts disappear if their partial derivative is zero.

p. 23, Exercise 3.1. "RE^m to $\mathbb{R}^n$" → "$\mathbb{R}^m$ to $\mathbb{R}^n$".

p. 33, Problem 3.2.1. Append the sentence
"Then for fixed x, the differential is the linear transformation $h \mapsto \mathbf{f}'(x)h$."

p. 33, Problem 3.2.3. $(\rho, \theta, \phi) \to (\rho, \phi, \theta)$

p. 34, first and second displayed equations should read

$$\begin{aligned} \mathbf{x}, \mathbf{h} \in \mathbb{R}^n &\Rightarrow (\mathbf{g} \circ \mathbf{f})(\mathbf{x}) \in \mathbb{R}^p \Rightarrow (\mathbf{g} \circ \mathbf{f})'_{\mathbf{x}}(\mathbf{h}) \in \mathbb{R}^p, \\ \mathbf{x}, \mathbf{h} \in \mathbb{R}^n &\Rightarrow \mathbf{f}'_{\mathbf{x}}(\mathbf{h}) \in \mathbb{R}^m \Rightarrow (\mathbf{g}'_{\mathbf{f}(\mathbf{x})} \circ \mathbf{f}'_{\mathbf{x}})(\mathbf{h}) \in \mathbb{R}^p. \end{aligned}$$

p. 34. Line should read $\overset{4}{=} \ldots + \left[\mathbf{g}'\big(\mathbf{R}(\mathbf{h})\big)\,|\mathbf{h}| + \mathbf{S}(\mathbf{k_h})|\mathbf{k_h}|\right]$.

p. 34. Replace the end of the page with the following:
The added phrase "divided by $|\mathbf{h}|$" is the reason for the changes.
To finish, we show that the term in brackets above,
divided by $|\mathbf{h}|$, approaches zero with $|\mathbf{h}|$. First, using the continuity of $\mathbf{g}'$ (Theorem 2.10),

$$\lim_{\mathbf{h}\to\mathbf{0}} \mathbf{g}'\big(\mathbf{R}(\mathbf{h})\big) = \mathbf{g}'\big(\lim_{\mathbf{h}\to\mathbf{0}} \mathbf{R}(\mathbf{h})\big) = \mathbf{g}'(\mathbf{0}) = \mathbf{0}.$$

Second, with $|\mathbf{f}'|_{\mathcal{O}}$ the operator norm of $\mathbf{f}'$,

$$|\mathbf{k_h}| \le |\mathbf{f}'(\mathbf{h})| + |\mathbf{R}(\mathbf{h})|\,|\mathbf{h}| \le |\mathbf{f}'|_{\mathcal{O}}|\mathbf{h}| + |\mathbf{R}(\mathbf{h})|\,|\mathbf{h}|.$$

Thus, since $(\mathbf{h}\to\mathbf{0}) \Rightarrow (\mathbf{k_h}\to\mathbf{0}) \Rightarrow (\mathbf{S}(\mathbf{k_h})\to\mathbf{0})$, $\lim_{\mathbf{h}\to\mathbf{0}} |\mathbf{S}(\mathbf{k_h})||\mathbf{k_h}|/|\mathbf{h}| = \mathbf{0}$.

p. 37, statement of Theorem 3.16.
"Then the inverse function $(\mathbf{f}'_{\mathbf{a}})^{-1}$" → "Then the inverse function $\mathbf{f}^{-1}$".

p. 40, following Theorem 3.13: "In other words, $\partial_{\mathbf{h}}\mathbf{f}$ is linear in both $\mathbf{h}$ and $\mathbf{f}$."

p. 40, Problem 3.3.5. The variable names I used lead to confusion. Change to $\mathbf{f}(x, y) = (x\cos y, x\sin y)$. And add "(All coordinates are cartesian.)"

p. 40, Problem 3.3.3. "continuity of f at x." → "continuity of f at $\mathbf{x}$."

p. 41, Statement of Theorem 3.19.
"has a differentiable inverse" → "has a continuously differentiable inverse".

p. 42, third line. Remove " there is a neighborhood of each y_i in which".

p. 45, Exercise 3.6.1b. "Determine $\partial\rho/\partial z$."

p. 48, below Eq. (4.3). "of higher dimension" → "in higher dimensions".

pp. 49-55. Equation (4.4) in Section 4.1 established the notation of m-dimensional manifolds M as subsets of $\mathbb{R}^n$. However, Sections 4.2 and 4.3, while mostly internally consistent, are inconsistent with this notation. The following changes remove the inconsistency:

- p. 49, 2^{nd} paragraph: "a curve C in $\mathbb{R}^n$."
- p. 50, Theorem 4.2: m to n.
- p. 51, Theorem 4.4: $\mathbb{R}^m$ to $\mathbb{R}^n$.
- p. 52, Theorem: $\mathbb{R}^m$ to $\mathbb{R}^n$, $\mathbb{R}^n$ to $\mathbb{R}^{\bar{n}}$.
- p. 53, First sentence and left part of Figure 4.5: $\mathbb{R}^m$ to $\mathbb{R}^n$.
- p. 53, Definition 4.5: $\mathbb{R}^m$ to $\mathbb{R}^n$.
- p. 53, line following Eq. (4.11): m to n.
- p. 55, Theorem: $\mathbb{R}^m$ to $\mathbb{R}^n$, $\mathbb{R}^n$ to $\mathbb{R}^{\bar{n}}$.

p. 49, Paragraph 2. "Let $\mathbf{x}(t)$ parameterize a curve C.
Then $\mathbf{x}$ has a nonzero differential (p. 49). By Problem 3.2.1 it is ..."

p. 50, line after Theorem 4.3. "Definition 4.5" → "Eq. (4.5)"

p. 52, Theorem. "$\mathbf{f}'_\mathbf{p}$ is one-to-one." → "$\mathbf{f}'_\mathbf{p}$ restricted to $\mathsf{T}_\mathbf{p}$ is one-to-one."

p. 52, Problem 4.2.3.
a. Show that components of $\boldsymbol{\Omega}$ must have grade 2 or 3.
b. Show that components of $\boldsymbol{\Omega}$ must have grade 0 or 2.

p. 53, Eq. (4.10). "$\lim_{\mathbf{h}\to\mathbf{0}}$" → "$\lim_{h\to 0}$"

p. 53, sentence below Definition 4.5. "Recall that the differential $\mathbf{x}'_\mathbf{q}$ is one-to-one and maps linearly independent vectors ... "

p. 55, Theorem. "$\mathbf{f}'_\mathbf{p}$ is one-to-one." → "$\mathbf{f}'_\mathbf{p}$ restricted to $\mathsf{T}_\mathbf{p}$ is one-to-one."

p. 56, Problem 4.3.1b. $\mathbf{x}_u \wedge \mathbf{x}_v \to \mathbf{x}_\phi \wedge \mathbf{x}_\theta$.

p. 57, Definition 5.1. "Let M be a manifold in $\mathbb{R}^n$. A *field* on M is a function defined on M whose values are in $\mathbb{G}^n$."

p. 62, Problem 5.2.5. Change to $\boldsymbol{\nabla}\cdot\big(\mathbf{x}f(|\mathbf{x}|)\big) = nf(|\mathbf{x}|) + |\mathbf{x}|f'(|\mathbf{x}|)$.

p. 64, Problem 5.2.11b. Remove the word "both".

p. 67, Exercise 5.14a. $\boldsymbol{\nabla}\cdot\mathbf{f} = \partial_1 f_1 + \partial_2 f_2$.

p. 69, Exercise 5.17b. Printing 1 only. Replace "The bases $\{\boldsymbol{w}_r(r,\theta)\}$ and $\{\boldsymbol{w}_\theta(r,\theta)\}$ are not in general orthogonal" with "In general, neither $\{\boldsymbol{w}_r(r,\theta)\}$ nor $\{\boldsymbol{w}_\theta(r,\theta)\}$ is an orthogonal basis."

p. 70, Exercise 5.33. "*Hint*: For Part (a) use Exercises 5.30 and 5.31."

p. 71, Figure 5.11. "$\mathbf{x}(c_1, c_2, c_3)$" → "$\mathbf{x}(c_1, c_2, u_3)$"

p. 71, Paragraph 4. In general: (i). Each basis vector $\mathbf{x}^k$ is orthogonal to the surface formed by fixing the coordinate u_k. (ii). Each basis vector $\mathbf{x}_j$ is tangent to the curve which is the intersection of the two surfaces formed by fixing in turn the coordinates other than u_j.

p. 72, formula for $\boldsymbol{\nabla} f$ in cylindrical coordinates. "$r^{-1}\partial f_\theta$" → "$r^{-1}\partial_\theta f$".

p. 74, Problem 5.5.2. Better: *Hint*: If $\mathbf{B}$ is a blade, then $\mathbf{B}^{-1} = \mathbf{B}/\mathbf{B}^2$, where $\mathbf{B}^2$ is a scalar.

p. 74, Problem 5.4.5. Delete. Renumber Problems 5.4.6-5.4.8 to 5.4.5-5.4.7.

p. 75, line 3. "manifolds in $\mathbb{R}^m$" $\to$ "manifolds in $\mathbb{R}^n$".

p. 76, Problem 5.6.1. "to the unit sphere" $\to$ "in $\mathbb{R}^2$"
"$\mathbf{D} = \mathsf{P_T}(\boldsymbol{\partial})$" $\to$ "$\mathbf{D}F = \mathsf{P_T}(\boldsymbol{\partial}F)$".

p. 81, Problem 6.1.4. $\begin{bmatrix} m \\ b \end{bmatrix} \to \begin{bmatrix} \bar{m} \\ \bar{b} \end{bmatrix}$.

p. 82, first paragraph "substitute the result in $x^2/8 + y^2/2$" $\to$
"substitute the result in xy"

p. 82, Theorem 6.7, proof.
"Let $\mathbf{x}(\boldsymbol{\xi})$ parameterize ... $\mathbf{x}(t) = \mathbf{x}(\boldsymbol{\xi}(t))$ parameterize a curve " $\to$
"Let $\mathbf{x}(t)$ be a parameterized curve".

p. 83, Problem 6.2.3. "on the triangle" $\to$ "inside the triangle".
Ans. Max 20, Min 4.

p. 89, last line of the displayed equation in the proof of Theorem 7.3
$\overset{|P|\to 0}{=}$ to $\overset{|P|\to 0}{\to}$.

p. 89. Replace the bulleted points with

- A definite integral of f, a number. It is the limit of sums (Definition 7.1). The number can represent areas, masses, etc.
- An indefinite integral of f, a function. It is an F such that $F' = f$.

p. 95, Definition 7.8. "tangent line to S" $\to$ "tangent line to C".

p. 117, Problem 8.2.3. Should read $x^2/a^2 + y^2/b^2 + z^2/c^2 \le 1$.

p. 119, first two paragraphs. $f \to F$ in the integrands.

p. 122, Corollary 9.4, statement. "$S \subset \mathbb{R}^n$" $\to$ "$S \subset \mathbb{R}^3$."

p. 130, Exercise 10.2.
"defined on the boundary" $\to$ "defined on M and ∂M".

p. 133, toward bottom. "which is wanting in the definition given by Eq. (5.3)."

p. 134, Problem 10.2.4b. Change vector field $\mathbf{f}$ to scalar field f. Drop Part c.

p. 136, Corollary 10.5, Proof. "Step (2) uses LAGA Theorem 6.30c and $d\mathbf{S}^* = d\boldsymbol{\sigma}$."
$\to$ "Step (2) uses LAGA Theorem 6.30c."

p. 138, toward bottom.
"which is wanting in the definition given by Eq. (5.3)."

p. 141, top. "manifolds of arbitrary dimension." $\to$ "$\mathbb{R}^m$."

p. 142, Theorem 10.10, statement. Serious error!
"Let M be an m-dimensional manifold." $\to$ "Let M be a bounded open set in $\mathbb{R}^m$."

$$F(\mathbf{x}_0) = \frac{(-1)^{m+1}}{\Omega_m \mathbf{I}_m} \oint_{\partial M} \frac{\mathbf{x} - \mathbf{x}_0}{|\mathbf{x} - \mathbf{x}_0|^m}\, d^{m-1}\mathbf{x}\, F(\mathbf{x}),$$

Corollary 10.11, statement.
"Let M be a manifold." $\to$ "Let M be a bounded open set in $\mathbb{R}^m$."

p. 148, Theorem 11.5, first sentence of statement.
Change to "Let $\mathbf{x}(s)$ and $\bar{\mathbf{x}}(s)$, $0 \le s \le L$, parameterize curves C and $\bar{C}$."

p. 150, Theorem 11.8, proof. $\ell(C) = \int_{[a,b]} \left|\mathbf{x}'\left(u_1(t), u_2(t)\right)\right| dt$

$$|\mathbf{x}'(u_1, u_2)|^2 = \mathbf{x}'(u_1, u_2) \cdot \mathbf{x}'(u_1, u_2) \overset{2}{=}$$

p. 152, Theorem 5.25, proof. $\mathbf{f}'(\mathbf{h}) \to \mathbf{f}'_{\mathbf{p}}(\mathbf{h})$.

p. 152, Exercise 11.24a. Show that the metric $G(r, \theta) = \left[\begin{smallmatrix} r & 0 \\ 0 & 1 \end{smallmatrix}\right]$.

p. 161, Figure 11.3. Remove the hat on $\mathbf{p}_1$ and $\mathbf{p}_2$.

p. 163, Standard Terminology. "The metric G (Eq. (11.5))" $\to$
"The expression $ds^2 = g_{ij} du_i du_j$ (Eq. (11.7))".

p. 172, Differentiation entry. "`print diff(diff(x**2,x),y)`" $\to$
"`print diff(diff(y*x**2,x),y)`".

p. 172, Jacobian entry. Redo:
Jacobian. Let X be an $m \times 1$ matrix of m variables. Let Y be an $n \times 1$ matrix of functions of the m variables. These define a function $\mathbf{f}\colon X \in \mathbb{R}^m \mapsto Y \in \mathbb{R}^n$. Then `Y.jacobian(X)` is the $n \times m$ matrix of $\mathbf{f}'_{\mathbf{x}}$, the differential of $\mathbf{f}$.

```
r, theta = symbols('r theta')
X = Matrix([r, theta])
Y = Matrix([r*cos(theta), r*sin(theta)])
print Y.jacobian(X)         # Print 2 × 2 Jacobian matrix.
print Y.jacobian(X).det()   # Print Jacobian determinant (only if m = n).
```

Sometimes you want to differentiate `Y` only with respect to some of the variables in `X`, for example when applying Eq. (3.24). Then include only those variables in `X`. For example, using `X = Matrix([r])` in the example above produces the 2×1 matrix $\left[\begin{smallmatrix} \cos\theta \\ \sin\theta \end{smallmatrix}\right]$.

p. 172, **Iterated Integrals** entry.
"`make_symbols('x y')`" $\to$ "`x, y = symbols('x y')`".

p. 174, **Reciprocal Basis** entry. Drop everything after the first sentence.

p. 175. Change to
Compute the vector derivative $\boldsymbol{\partial}\mathbf{f}$, divergence $\boldsymbol{\partial} \cdot \mathbf{f}$, curl $\boldsymbol{\partial} \wedge \mathbf{f}$:
`M.grad * f, M.grad < f, M.grad ^ f.`